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CONSTRUCTION OF NEW STRUCTURE OF 4-REGULAR PLANAR GRAPH AND ITS APPLICATIONIN LATTICE STRUCTURE

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ABSTRACT

The modularity of a graph is a parameter that measures its structure.In this article we have construct a new structure 4 regular planar graph G(2s+6, 3s+t). An application has been focused on lattice structure of different crystal structure.

Keywords: Regular planar graph, Cluster graph, lattice structure, pattern, crystal.

INTRODUCTION

A graph embedded in the sphere without edge crossings is a planar graph. We do not distinguish an outer face. On account of Euler's formula, a simple planar graph has average degree less than 6, and therefore a regular simple planar graph can have degree at most 5. However, although much is known about the structure of 3-regular and 4-regular simple planar graphs, little is known about the 5-regular case. Similarly, there is a large literature on planar graphs with every face of size 3 (that is, each face bounded by 3 edges), or every face of size 4, but very little for every face of size 5. The latter are planar pentangulations. Various researcher are working on Regular planar graph. Hasheminezhad Mahdieh et al [1] have been described that the 5-regular simple planar graphs can all be obtained from an elementary family of starting graphs by repeatedly applying a few local expansion operations. The proof uses an amalgam of theory and computation. By incorporating the recursion into the canonical construction path method of isomorph rejection, a generator of non-isomorphic embedded 5-regular planar graphs is obtained with time complexity $O(n^2)$ per isomorphism class. A similar result is obtained for simple planar pentangulations with minimum degree 2. Gardemann Thomas et al [2]have beenconstructa new families of integral graphs by considering complete products, unions and point identifications of complete graphs and complete bipartite graphs. They also find a relation between arithmetic series and the integrality of complete products. Exoo Geoffrey et al[3] have been investigate the properties of the resulting graphs in the context of cages and construct families of small graphs using geometric graphs, Paley graphs, and techniques from the theory of Cayley maps. Bekos A Michael[4] has been proposed that affirmatively answer Lov´asz's conjecture, if G is 3-connected, and demonstrate an infinite class of connected 4-regular planar graphs which are not 3-connected and do not admit a realization as a system of circles.Yangyan Gu et al [5] have been proved that every nn-vertex planar graph has a 3-degenerate induced subgraph of order at least $3n/4$3n/4.Islam at al [6] have been proposed the construction of a structure of the 4-regular planar graphs for G(2m+2,4m+4) where m≥2. They have define two theorems on odd regions and total regions of 4-regular planar graphs and also prove it. Finally an application is given in region base map coloring and GSM network coloring. Islam et al[7] have been discuss about of a new cluster graph G(2n+6, 3n+m). They find minimum vertex cover of the graph G(2n+6, 3n+m). In addition to this, they also deloved an algorithm to find out minimum vertex cover of any graph. Finally an application of minimum vertex cover has been focused on sensor network.Vijaya M et al[8] have been introduced picture fuzzy soft graph, lower and upper truncation of picture fuzzy soft graph. They also studied truncation of subdivision picture fuzzy soft graph and truncation of strong picture fuzzy soft graph.

OUR WORK-

Theoretical Investigation-

In this article we have consider a graph G(V,E)=G(2s+6, 3s+t). Here V=2s+6 is the set of vertices and E=3s+t is the set of edges and s and t are the arbitrary values. Recently, Islam and Haloi [7] introduced a new cluster graph G(2s+6, 3s+t) and find minimum vertex cover of the graph G(2x+6, 3x+y).They also developed an algorithm, theorem an application of minimum vertex cover.

In this article we have construct a new cluster 4 –regular planar graph according the incremented values of s and t in the following way-

Construction of 4-Regular Planar Graph:

Let G be a graph having (2s+6) vertices and (3s+t) edges for s=1,and t=13, G contains eight vertices $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and 16 edges. Let us join these eight vertices by 16 edges to construct 4 Regular Planar graph.

$\propto (v_i) = v_{i+1}\ for\ \ 1\leq i \leq 7$

v_{i-3} i=4

v_{i-3} i=8

$\propto (v_i) = v_{i+4}\ \ 1\leq i \leq 4$

$\propto (v_i) = v_{i+5}\ \ 2\leq i \leq 2$

$v_1$for i=3

$v_8$for i=6

Then we have the edge set $\{v_1 v_2, v_2 v_3, v_3,v_4,v_4 v_1,v_5 v_6,v_6v_7, v_7 v_8 ,v_8 v_5,v_1v_5,v_2v_6,v_3 v_7,v_4v_8,v_2v_5,v_1 v_3,v_4v_7,v_8v_6\}$ and we obtain a graph [Figure-1], which is planar and regular of degree four.

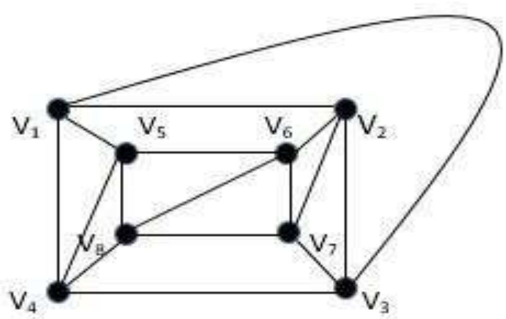

Figure- 4 Regular planar graph

Now for s=2 and t=14 then the graph G contains 10 vertices $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ and 20 edges. Let us join these 10 vertices by 20 edges to construct 4 Regular Planar graph.

$\propto (v_I) = v_{i+1}\ for\ \ 1\leq i \leq 9$

$=\qquad v_{i-4}$ i=5

v_{i-4} i=10

$\propto (v_i) = v_{i+5}\ \ 1\leq i \leq 5$

$\propto (v_i) = \qquad v_{i+6}\ \ 2\leq i \leq 4$, i≠3

$v_1$for i=3

$\propto (v_i) = \qquad v_9$for i=7

Then we have the edge set $\{v_1 v_2, v_2 v_3, v_3,v_4,v_4 v_5,v_5 v_1, v_6v_7, v_7v_8, v_8 v_9 ,v_9 v_{10},v_{10} v_6, v_1v_6, v_2v_7,v_3 v_8, v_4v_9, v_5v_{10}, v_2v_6,v_1 v_3, v_4v_8,v_5v_9,v_7v_{10}\}$ and we obtain a graph [Figure-2], which is planar and regular of degree four.

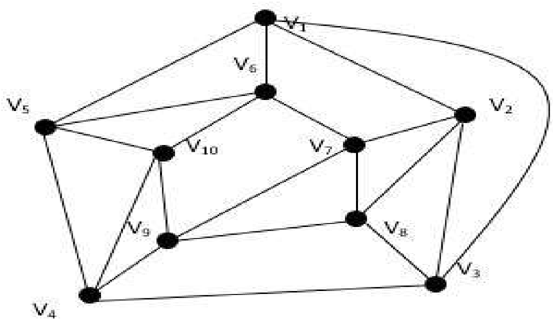

Figure-2- 4-regular planar graph.

For the graph G having 2s+6 number of vertices and 3s+t number of edges for s≥1 and t ≥13, we define ∝: VG→VG such that

$\propto(\ ) = \quad v_{i+1}\, for\ 1 \le i \le 2s+5$

$v_1 for V_3$

$\propto(v_i) = v_{s+3+i} for\ 1 \le i \le s+3$

$\propto(\ ) = v_{s+4+i} for\ 2 \le i \le s+1, i\neq3$

$\propto(\ ) = v_{s+4+i} for\ 4 \le i \le s+2,\ s\geq2$

$\propto(v_1) = v_{i+3} for$ i=s

$\propto(v_i) = v_{i+s+2}$ when i=5,s=1 and s=2,i=6 i.e alternative increment of s and i value.

$\propto(\ ) = v_{i+2} for$ i=s+5

From the above generalize formulae we can construct the above structure 4-Regular Planar Graph for the graph G (V, E) =G (2s+6, 3s+t). The generalize formulae also construct a planar graph which satisfy the planarity condition i.e e < 3n-6, where e= no of edges contain by the graph and n= no of vertices contain by the graph.

APPLICATION

On a finite set of a combinatorial objects, distributive lattices are attractive and well understood structures and it is always nice to identify a distributive lattice. As an example we take a crystal structure as in fig-3.

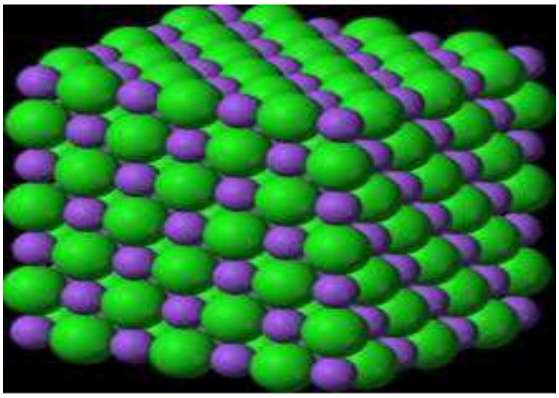

Figure-3- Crystal structure

A lattice is a series of points that are arranged in a distinct pattern. A crystal lattice structure is like to a lattice, but instead of points, it is composed of a series of atoms. A crystal lattice is typically arranged in some sort of symmetrical geometric shape, with each vertex representing an atom.

Many solids possess a lattice structure because the total intermolecular energy is lowest when the atoms are arranged in these types of geometrical shapes.

There are many different types of lattice formations that a crystal structure can have, including:

- Face-centered cubic is a common lattice structure for copper, silver, platinum, nickel and aluminum.
- Body-centered cubic is a common lattice structure for tantalum, iron, chromium and molybdenum.
- Hexagonal close-packed lattice structures can be found in magnesium, cobalt, zinc and titanium.
- Tetragonal lattice structure is found in some types of steel.

Steel is a material that can have many different types of crystal lattice structures. In some forms at room temperature, it has a body-centered cubic composition. When it is heated to a certain temperature, its crystal structure turns into a face-centered cubic form. If it is cooled rapidly from a heated state and if it has a high amount of carbon in its chemical composition then it will have a body-centered tetragonal crystal lattice structure.

If we represent the above crystal structure as unit cell and lattice structure, the structure will be 4 –regular planar graph which represent in Fig- 4

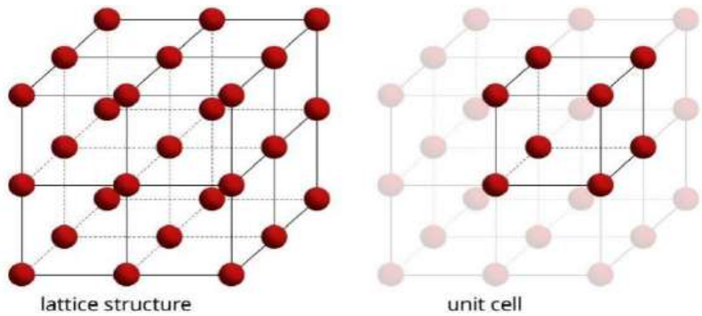

Figure-4 – Lattice structure and unit cell structure of the crystal.

Through the graph structure of the graph G (V, E) =G (2s+6, 3s+t) we can construct 4 regular planar graph which is resembles with the above crystal structure. In the graph atoms are represented as vertices and connections of atoms are represented as edges. In the fig-4 we have seen that the end vertices degree are connected with three edges and on edge is hidden connected. So the end vertices degree simple look like three degree.The graph structure of Fig-2 and Fig -3 use as crystal structure of different crystal as well as also use as lattice structure. There are lot of lattice structure like salt, Schnyder woods which resembles in our graph structure which represents a 4 regular planar graph. Therefore the graph structures which is constructed by the above generalize formulae we can represent as different crystal lattice structure.

CONCLUSION

This paper is proposes a manual assisting method to extract knowledge to construct the 4 regular planar graph. From the above generalize formulae one can construct the 4 regular planar graph G (V, E) =G (2s+6, 3s+t) where $s \geq 1$ and $t \geq 13$. The graph structure can resembles with the crystal structure as well as also resembles with the lattice structure of crystal.

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